

## Dealing with Wind Power Forecast Uncertainty

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**ABSTRACT.** In power systems with a large integration of wind power, setting the adequate operating reserve levels is one of the main concerns of System Operators (SO). The integration of large shares of wind generation in power systems led to the development of forecasting systems, able to produce probabilistic forecasts, but management tools able to use those forecasts to help making operational decisions are still needed. In this paper, a risk evaluation perspective is used, showing that it is possible to describe the consequences of each possible reserve level through a set of risk indices useful for decision making. Then, decision-aid methods are used for helping the decision maker in setting the operating reserve requirements.

**KEYWORDS.** Multicriteria; decision issues; uncertainty; risk measures; operating reserve; wind power forecast.

### 1. INTRODUCTION

Electric power systems are variable in nature regarding both demand and supply. The combination of generation and load uncertainty requires a corrective management of the power system to maintain the supply-demand balance. Therefore, the System Operator (SO) mobilizes additional generation capacity, frequently known as operating reserve, to hedge the risk of losing load. Traditionally, risk represents the probability of failing to satisfy the load in the short-term, as the result of loss of generation due to forced outages and load forecast errors. When the level of wind power generation in the system becomes important, the impact of wind power uncertainty on the load-generation balance becomes critical. Wind power becomes the third source of risk. Hence, the operating reserve acts as a provision against outages and forecast errors. Only secondary (or spinning) and fast tertiary reserves (or non-spinning reserve available within a short-period) are considered as operating reserve controlled by the SO.

In general, many SO still prefer to use deterministic criteria for defining operating reserve requirements. These deterministic criteria normally are the loss of the largest generation unit or an empirical rule such as the UCTE (Union for the Coordination of the Transmission of Electricity) rule (UCTE, 2004). However, these deterministic approaches do not measure the risk, and it may happen that, in some circumstances, complex risky situations are not covered. In the research literature there are more elaborated rules that try to include the uncertainty of wind power forecast. One example is a simple probabilistic model that computes the square root of the sum of the squares of the standard deviations of hourly wind power and load forecast errors (Holtinen, 2004). The reserve is defined to cover all variations contained within  $\pm 3\sigma$  of the total system forecast error, which means that 99.74% of variations are covered assuming a Gaussian distribution for the two variables.

The structure of this paper is as follows: section 2 describes a probabilistic approach (Matos and Bessa, 2009) to build the probability mass function (*pmf*) of the generation margin (amount by which the

available generating capacity exceeds the system load) that results from conventional generation availability, load and wind probabilistic forecasts. The consequences of each possible reserve level are described through a set of risk indices useful for decision making. This case is not favorable to the application of utility functions, since the Archimedean axiom (Chankong and Haimes, 1983) seems not to be satisfied, due to the catastrophic consequences of not serving the load in these circumstances, while at the same time all the situations with positive generation margin are more or less indifferent. Section 3 presents different decision-aid methods to incorporate the preferences of the Decision Maker (DM). In section 4, an illustrative example is presented, where results from different decision making methods are discussed. The conclusions complete the paper.

## 2. PROBABILISTIC MODEL

### 2.1. Uncertainty Representation

The load uncertainty is modeled through a Gaussian distribution with a given standard deviation and zero mean (Allan and Billinton, 1984).

The probability mass function (*pmf*) of the conventional generation is analogous to the discrete probability distribution of all possible combinations of forced generation failures, known in the literature as the capacity outage probability table (COPT) (Allan and Billinton, 1984). The process of building a COPT is a convolution between all generation units with two possible states (on or off) and a probability associated to each state ( $1-p_i$  and  $p_i$  for each unit  $i$ ) (Allan et al., 1981).

The uncertainty in wind power is related with the inherent error in wind power forecast. As a result from the ANEMOS project (<http://anemos.cma.fr/>), different methods to estimate the uncertainty of deterministic (or point) forecasts were developed (Pinson et al., 2007). The uncertainty representation in this paper is non-parametric and represented by a set of quantiles.

### 2.2. System Generation Margin Probability Distribution

The approach consists in computing the system generation margin (as  $M=C+W-L$ ), for each look-ahead time step, by a convolution between the conventional generation distribution (C), the wind generation distribution (W) and the load (L). This convolution is computed in an efficient way with the Fast Fourier Transform (FFT) procedure.

The system generation margin distribution is a discrete probability distribution for each look-ahead time step, represented by its *pmf*; as depicted in Fig. 1 where the negative values of the margin are marked, since they correspond to loss of load situations in the absence of reserve. Moreover, for a specific level of operating reserve  $R$ , the distribution of  $M+R$  describes the probability of the reserve being sufficient to cover the deficit of generation. Fig. 2 shows the effect of setting a reserve level of 800 MW in the same situation of Fig. 1. This additional capacity (as reserve) means shifting the Fig. 1 *pmf* to the right by the amount of the reserve 800 MW.

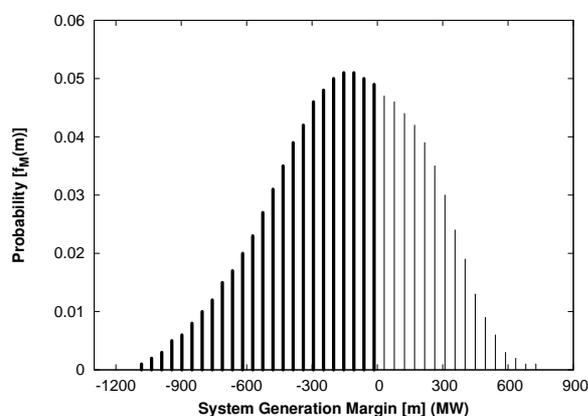


Figure 1. *pmf* of the generation margin for a specific look-ahead time step.

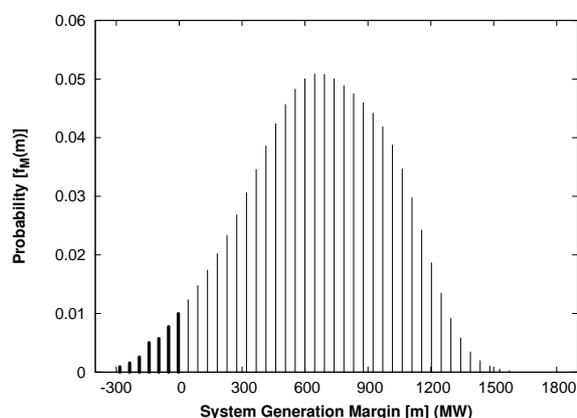


Figure 2. *pmf* of the generation margin for a operating reserve of 800 MW.

### 2.3. Risk Attributes Related to the Amount of Loss of Load

At this point, risk attributes related to the amount of loss of load, and meaningful for the Decision Maker (DM), can be computed from the system generation margin. Following the approach described in (Matos, 2007), the idea is to characterize the system generation margin distribution from Fig. 1 or 2 by a set of risk attributes, in order to give meaningful information to the DM about the impact of a potential reserve level.

The classical measures in reliability (Allan and Billinton, 1984) can be calculated from the system generation margin distribution, e.g. loss of load probability (LOLP), loss of load expectation (LOLE) or the expected power not supplied (EPNS). LOLP represents the probability of the system load exceeding the available generation (negative margin) in a given look-ahead time step and is given by  $\sum_{m \leq 0} f_M(m)$ , where  $f_M(m)$  is the pmf of the margin. For instance, in the situation depicted in Fig. 1 (without any reserve), we would have a LOLP of 0.64, but, with the 800 MW reserve LOLP reduces to only 0.033. This means that if for the time step period (for instance 60 minutes) we randomly choose a minute, the probability of having a loss of load in that minute is 0.64 without reserve and 0.033 with reserve. LOLE represents the expected number of time units in a given period that the load exceeds the available generation and corresponds to  $LOLP \cdot T$ , where  $T$  is the period under consideration. For Fig. 1, and assuming a look-ahead time step of 60 min, the LOLE is 38.3 min/hr, and in Fig. 2 is 2 min/hr. The EPNS represents the expected power which can not be supplied, in a given period, due to insufficiency of generation and is given by  $\sum_{m \leq 0} |m \cdot f_M(m)|$ . EPNS for Fig. 1 is 212.3 MW and for Fig. 2 is 2.82 MW. Also other measures of reliability such as the conditional expected value of loss of load (XLLOL) can be computed (Day et al., 1972). XLLOL represents the average loss of load conditioned when it happens and is given by  $E\{m | m \leq 0\} = \sum_{m \leq 0} |m \cdot f_M(m)| / \sum_{m \leq 0} f_M(m)$ . This index is the ratio between the EPNS and LOLP and therefore for Fig. 1 is 332.3 MW and for Fig. 2 is 84.2 MW.

Other measures of risk borrowed from the financial area, such as the value-at-risk (VaR) and conditional value-at-risk (cVaR) can also be computed (Wu et al., 2008). The  $VaR_\alpha$  of the system generation margin can be defined as  $\inf\{x \in \mathcal{R} : f_M(m \leq x) \leq 1 - \alpha\}$ , where  $\alpha$  is the confidence level. In words, this is the maximum value  $x$  for which the probability that the loss of load (negative values of the margin)  $m$  exceeds  $x$  is not larger than  $(1-\alpha)$ . For instance, the  $VaR$  with  $\alpha=90\%$  for Fig. 1 is -618 MW, meaning that we are 90% sure the loss of load will not be greater than 618 MW (a very unsafe value). On other hand, the  $VaR_{90\%}$  in Fig. 2 is positive and equals 181 MW, meaning that with 90% sure there is extra capacity in the system. The  $cVaR_\alpha$  can be defined as  $E\{m | m \leq VaR_\alpha\}$  and represents the expected generation margin when the margin is below the  $VaR_\alpha$  value. The value of  $cVaR_{90\%}$  in Fig. 1 is -748.8 MW and in Fig. 2 is 3.8 MW.

Risk measures, related with surplus of generation (downward reserve), can also be computed from the positive part of the system generation margin distribution. These risk measures are analogous to the ones related with the loss of load, for example: the probability of wasting renewable energy.

### 2.4. Risk/Reserve Curves

As a result from the exercise depicted in Fig. 1 and 2 curves with reserve as a function of risk can be obtained. For instance, if we take the cumulative distribution of the system generation margin, a direct reading of LOLP and LOLE is possible for each reserve level.

This kind of risk/reserve curve (represented by points) is depicted in Fig. 3 for the risk measures LOLP, EPNS,  $VaR_{90\%}$  and XLLOL (for the illustrative example presented in Section 4). The LOLP and EPNS present a non-linear monotonic decrease with the increasing level of reserve. The behavior of XLLOL is slightly different because it corresponds to the ratio EPNS/LOLP; as we may see from Fig. 3, the two curves do not present the same decrease rate.

The  $VaR$  presents a linear relation with the reserve level, and the same happens with  $cVaR$ . The point in the curve where the  $VaR_{90\%}$  is zero is analogous to the reserve (in this case 510 MW) that corresponds to

a LOLP equal to  $1-\alpha$  ( $0=VaR_{1-LOLP}$ ), as depicted in Fig. 3. Moreover, this zero VaR point projected on the XLOL curve gives the  $cVaR_{90\%}$  value ( $XLOL=cVaR_{1-LOLP}$ ) when the reserve is equal to 510 MW (or equal to  $VaR_{90\%}$ ).

### 3. DECISION-AID METHODS

The preceding curves are the mathematical basis to set the level of operating reserve, since it allows the operator to evaluate the risk associated to a specific level of reserve. However it is convenient to formalize a decision-making phase based on the DM global perception and judgments, and the participation of DM during this process is absolute indispensable.

The same decision method is applied for each look-ahead time step and the output is the operating reserve level determined at a specific time instant for each look-ahead time (e.g. 1 hour) of the next day.

#### 3.1. Setting a Threshold for the Risk

The simplest approach consists in setting a reference threshold for the maximum acceptable risk. If the proposed reserve level leads to excessive risk then the reserve level must be increased until the actual risk value is lower than the threshold. For instance, in Fig. 3, in order to assure a LOLP not greater than 0.1, a reserve level of at least 512 MW would be necessary.

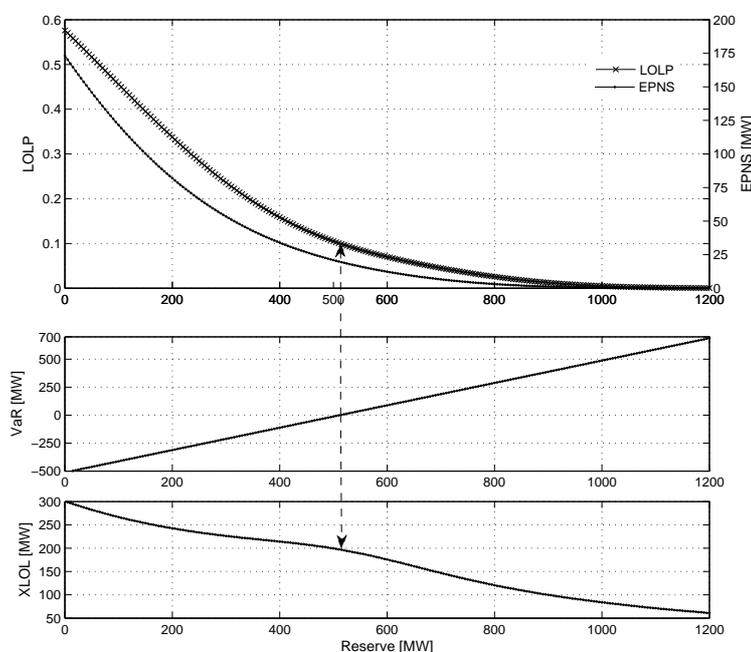


Figure 3. Risk/reserve curves of LOLP, EPNS,  $VaR_{90\%}$  and XLOL for a specific look-ahead time step.

#### 3.2. Multicriteria Analysis

Depending on the organization of the electricity market, the cost curve of the reserve can come from bids offered by the market agents or from a contract between the SO and a reserve provider. A multicriteria approach will be used to help the DM to balance risk and reserve cost.

##### 3.2.1. Equivalent Cost Approach

The equivalent cost approach uses a constant trade-off between reserve cost and an associated risk measure (e.g. EPNS). The trade-off  $\mu$  ( $=\Delta C/\Delta R$ ) is the rate at which the DM is ready to spend  $\Delta C$  units of cost in exchange for gaining  $\Delta R$  units in the risk criterion, while remaining indifferent between the two solutions. In this approach, after obtaining the trade-off from the DM, we just need to find the reserve level  $r$  that minimizes the equivalent cost  $Eq.Cost(r) = Cost(r) + \mu \cdot Risk(r)$ .

##### 3.2.2. Value Function Approach

An alternative approach is to use value functions capable of reflecting a more complex attitude (when

trade-offs may not be constant) of the DM towards the criterion. We will confine the approach to additive value functions functions (Keeney and Raiffa, 1976) (Chankong and Haimes, 1983), but more complex functions could be used. We will assume that additivity conditions for value functions with two attributes are met.

The approach consists in building an individual value function for each criterion, and then assessing weights to build the multi-attribute value function whose maximization leads to the preferred reserve level  $r$ . A possible multi-attribute value function for this problem would be:

$$v_{Cost,EPNS}(r) = k_{Cost} \cdot v_{Cost}(Cost(r)) + k_{EPNS} \cdot v_{EPNS}(EPNS(r)) \quad (1)$$

where  $v_{Cost}$  and  $v_{EPNS}$  are the individual value functions for the two criteria and  $k_{EPNS}$  and  $k_{Cost}$  are parameters, usually know as weights ( $k_{Cost} + k_{EPNS} = 1$ ). Only one risk measure was considered in this case, but extension to include more indices is straightforward, provided that the preferential independence is checked (Keeney and Raiffa, 1976) (Chankong and Haimes, 1983).

The shape of the value function reflects the variation in preference of the DM regarding the value of the corresponding attribute. The individual value function of the cost is usually linear, because the increase in DM satisfaction for a specific cost saving tends to be independent of the cost level. However, regarding EPNS a common situation corresponds to the DM appreciating more the decrease in the EENS when there is a high amount of energy not supplied than when there is a lower value of energy not supplied. Another DM, in contrast, could appreciate paying more for reducing risk when approaching lower levels of EPNS. The exponential function is capable of describing a behavior with variable preferences regarding risk (in this case through EPNS):

$$v_{EPNS}(r) = \frac{e^{b \cdot z(r)} - 1}{e^b - 1} \quad (2)$$

where  $z(r) = (EPNS^{\max} - EPNS(r)) / (EPNS^{\max} - EPNS^{\min})$ .

By changing parameter  $b$  it is possible to represent accurately the decision maker preference structure: negative values of  $b$  model the first attitude described earlier, while positive values correspond to the second attitude. The last step consists of determining the weights  $k_{Cost}$  and  $k_{EPNS}$ . It is essential to stress that these two parameters should not be asked explicitly to the DM based on relative importance of the criteria, since they also include scaling factors. When using predefined value functions, a single judgment of indifference is sufficient to calculate these parameters. An iterative process is used where artificial solutions successively are presented to the DM until an indifferent between two solutions is found (Keeney and Raiffa, 1976) (Chankong and Haimes, 1983).

#### 4. ILLUSTRATIVE EXAMPLE

The example used to demonstrate the methodology is based on the Portuguese power system, with 10395 MW of conventional generation (119 units) and 2742 MW of wind power. For illustrative purposes, a wind power probabilistic forecast similar to the ones produced by the ANEMOS platform is used. For simplicity, and without loss of generality, only look-ahead time 6:00 is analyzed. It is assumed that the Mean Absolute Percentage Error of the load forecast is 2% for a typical load curve of a weekday.

For comparison, the following rules are considered: A) UCTE rule ( $\sqrt{10 \cdot L_{\max} + 150^2} - 150$ ,  $L_{\max}$  is the peak load) plus capacity of the largest unit (235 MW); B)  $\varepsilon \cdot \sqrt{\sigma_L^2 + \sigma_W^2 + \sigma_C^2}$ , where  $\varepsilon$  is related with the confidence level and the three  $\sigma$  are the standard deviation of load (L), wind generation (W), and conventional generation (C) respectively. If for instance, the DM defines a LOLP of 0.01 the value of  $\varepsilon$  is defined to cover 98% of all variations of a Gaussian distribution,  $\varepsilon=2.33$ .

##### 4.1. Risk/Reserve Based Decisions

We first simulate a situation where three different thresholds for a risk measure (LOLP) were previously set by the DM. Furthermore, in order to analyze the consistency of each suggested reserve level a Monte Carlo simulation (20000 random samples from the forecasted distributions) was performed for hour 6:00. The results of the Monte Carlo and the reserve suggested by the methodology and rules A, B are

presented in Table 1.

The results obtained with the methodology presented in the paper are consistent with the risk threshold defined. Rule B presents a LOLP value higher than the max accepted level. The assumption of a Gaussian distribution for wind power uncertainty is not adequate, since it leads to a higher risk value than stipulated in the rule. This happens because of the asymmetry of forecast error distribution.

If the DM is comfortable with a risk of 1%, then rule A leads to an excessive risk. But in some cases the opposite situation could happen, and rule A would lead to excessive reserve. In fact, the rule does not measure risk or takes into account directly the level of wind power and its uncertainty.

Table 1. Results of the Monte Carlo simulation for hour 6:00.

		Methodology	Rule A	Rule B
LOLP=1%	Reserve [MW]	925	424	769
	Loss of load occurrences (%)	1%	14.5%	3.2%
LOLP=5%	Reserve [MW]	680	424	542
	Loss of load occurrences (%)	5.1%	14.5%	9%
LOLP=10%	Reserve [MW]	512	424	345
	Loss of load occurrences (%)	10%	14.5%	19%

The main drawback of the risk/reserve approach is the lack of intuitive interpretation of the risk indices and therefore may not be easy for a DM to define a reference risk threshold. Also since reserve cost is not taken into account, expensive additional generation may be required in order to maintain the risk below the threshold.

#### 4.2. Risk/Cost of Reserve Based Decisions

Each equivalent cost function value defines a family of linear indifference lines that describe the preference structure of the DM, their slope being the reference trade-off value. Fig. 6 depicts the indifference lines for three reference trade-off (25, 50 and 150 €/MW) and the risk/cost of reserve curve. For instance, for the trade-off 50 €/MW the preferred solution (x) is the one in the indifference line that has an equivalent cost of 4128 € and corresponds to a reserve of 440 MW, EPNS= 28.09 MW and reserve cost= 2724 €. Now, a different decision maker, more concerned with the level of EENS, sets a greater trade-off of 150 €/MW, and the preferred solution (z) will be different: more reserve (685 MW); less risk (EENS=7.3 MW) and higher cost (4413.36 €)

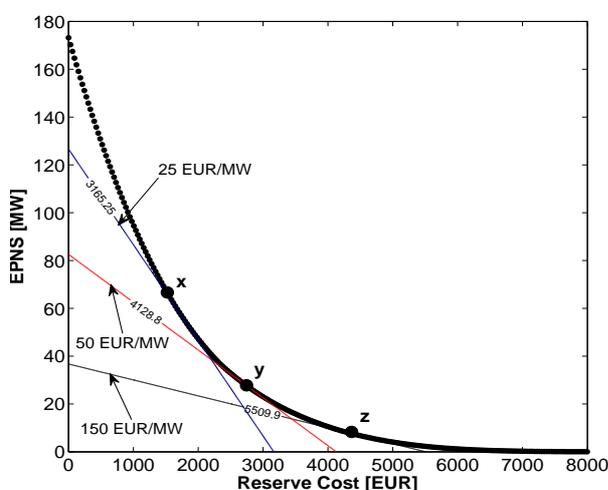


Figure 6. Indifference lines of the constant tradeoffs ( $\mu=25, 50$  and  $150$  €/MW).

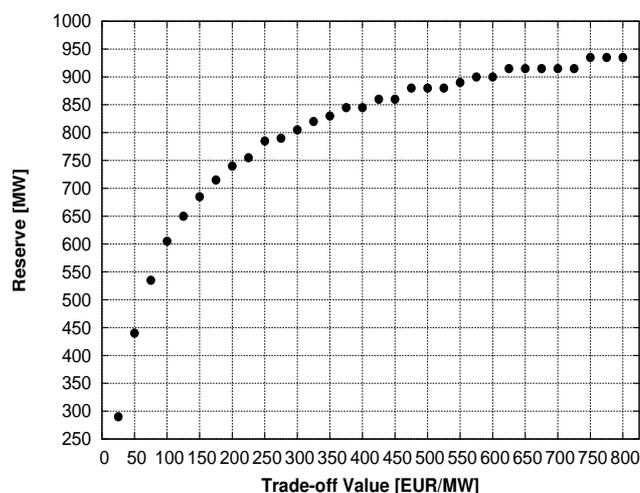


Figure 7. Reserve requirements obtained with different trade-off values.

Fig. 7 depicts the reserve obtained using a trade-off value ranging from 25 to 800 €/MW. As expected, as the trade-off value changes the preferred solution moves along the risk/cost of reserve curve. For lower trade-off values the preferred solutions are in the part of the risk/cost of reserve curve with higher slope and therefore variations of 25€/MW in the trade-off lead to higher variations in the reserve. As we move to zones with lower slope, the difference between the solutions becomes smaller. For instance, for trade-offs between 425 and 800 €/MW the reserve only increases from 860 to 935 MW (the same reserve value is obtained with trade-off values between 625 and 675 €/MW).

For more complex preference structures, a nonlinear value function can be used. Assuming (as explained before) the use of a normalized exponential function (2) for EPNS valuation, the DM only needs to specify a point from the value function in order to determine parameter  $b$ . One way consists in interacting with the DM to discover the midpoint  $x$  between  $EPNS_{max}$  (180 MW) and  $EPNS_{min}$  (0 MW), in terms of satisfaction. For instance, if the DM states that decreasing EPNS from 180 to 150 MW is equivalent to the decrease from 150 MW to 0, then  $v_{EPNS}(EPNS=150)=0.5$  and  $b=-4$ . With this exponential function and a linear function for the cost, we just need an additional indifference judgment to complete the model. So if the DM is indifferent between (5000,60)~(5500,50), we obtain the (nonlinear) indifference curves depicted in Fig. 8.

Since the trade-off is not constant, in the area of higher EPNS, a small decrease in EPNS offsets a large increase in reserve cost (the indifference curves are little sloped), because the DM is prepared to pay the necessary to avoid high values of EPNS. By contrast, in the low EPNS, a big decrease in EPNS offsets a small increase in reserve cost, since the values of EPNS are tolerable by the DM, so he will just pay something if a large decrease of EPNS happens. The preferred solution is in the indifference curve with value 0.9274 and corresponds to a reserve of 370 MW, EPNS of 39 MW and cost of 2259€.

Fig. 9 depicts the situation of a DM that increases his willingness to pay for reducing EPNS when approaching the lower levels of EPNS (becomes a full satisfaction seeker). In this case the trade-off in the first part of the curve is very low, and in the second is very high. The preferred solution lies in the indifference curve with value 0.6457 and corresponds to a reserve of 675 MW, EPNS of 7.8 MW and cost of 4341€.

Table 2 presents sensitivity analysis on the parameter  $b$ . As expected, when  $b$  is more negative the satisfaction of the DM increases more rapidly as the level of EPNS decreases. Consequently the solution obtained with  $b=-4$  has less reserve in comparison with  $b=-2$ . For positive values of  $b$  the satisfaction of the DM increases slowly in the zone with lower EPNS, therefore the reserve needs obtained with this solution are higher when compared with the ones obtained with negative values of  $b$  and increase with the value of  $b$ .

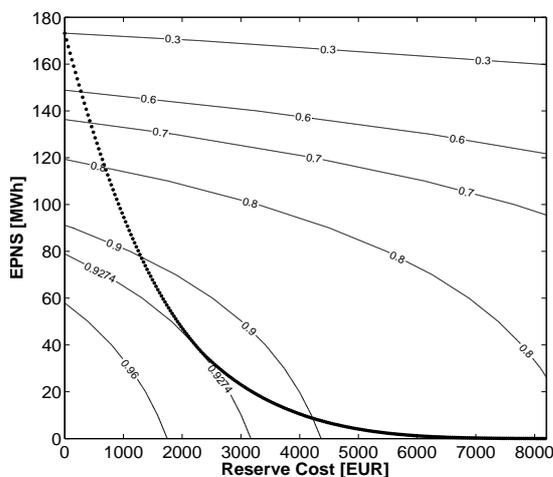


Figure 8. Indifference curves of exponential value function with  $b=-4$ .

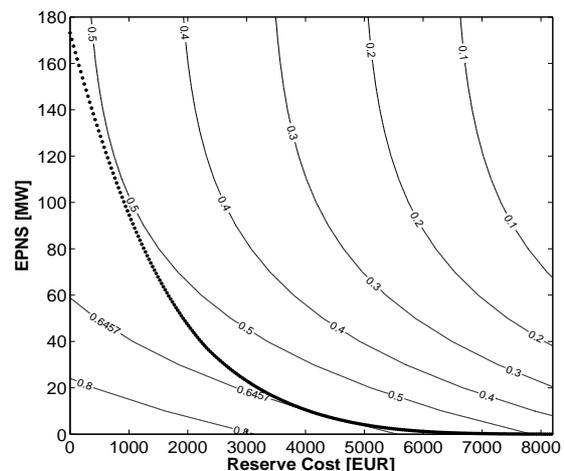


Figure 9. Indifference curves of exponential value function with  $b=4$ .

Table 2. Reserve obtained with exp. value function with different values of  $b$ , the indifference solution is (5000,60)~(5500,50)

b	-8	-4	-2	2	4	8
Reserve [MW]	265	370	385	535	675	845
EPNS [MW]	62.28	38.98	36.38	17.52	7.81	2.11
Cost [€]	1616.50	2258.82	2353.07	3366.15	4341.36	5571.67

## 5. CONCLUSIONS

This paper describes the decision-making issues associated with defining the operating reserve requirements, taking into account the uncertainties in generation (conventional and wind power) and load. Different risk measures are presented and analyzed to compute risk/reserve curves that give information on the impact of each possible reserve level. A relation between financial measures like value-at-risk and classical reliability measures like loss of load probability is established for this problem. Besides the risk characterization, the paper discusses different attitudes and preferences of the decision maker. For instance, by setting thresholds for the risk, the risk is maintained below the desired level; however, the balance between reserve cost and risk eliminates the possibility of excessive risk or buying reserve at all cost. In the nonlinear value function approach the exponential function due to its flexibility makes easier the interaction with the DM. Future work consists in demonstration and evaluation of the methodology at REN (Portuguese SO) control centre with real decision makers.

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## REFERENCES

- Allan RN, Billinton R, *Reliability evaluation of power systems*, Plenum Press, New York and London, 1984.
- Allan RN, Leite da Silva AM, Abu-Nasser A, Burchett RC, Discrete convolution in power system reliability, *IEEE Transactions on Reliability*, 30, 452-456, 1981.
- Chankong V, Haimes Y, *Multiobjective Decision Making Theory and Methodology*, Elsevier Science, New York, 1983.
- Day JT, Shortley PB, Skooglund JW, Expected Value of Generation Deficit: A Supplemental Measure of Power System Reliability, *IEEE Transactions on Power Apparatus and Systems*, 91, 2213-2223, 1972.
- Holtinen H, Impact of hourly wind power variations on the system operation in the Nordic countries, *Wind Energy*, 8, 197-218, 2004.
- Keeney R, Raiffa H, *Decision with Multiple Objectives: Preference and Value Tradeoffs*, Wiley, New York, 1976.
- Matos MA, Decision under risk as a multicriteria problem, *European Journal of Operational Research*, 181, 1516-1529, 2007.
- Matos MA, Bessa RJ, Operating Reserve Adequacy Evaluation using uncertainties of wind power forecast, Proceedings of the IEEE Power Tech Conference, Bucharest, Romania, June 28-July 2, 2009.
- Pinson P, Nielsen H, Madsen H, Lange M, Kariniotakis G, Methods for the estimation of the uncertainty of wind power forecasts, ANEMOS project deliverable report D3.1b, Informatics and Mathematical Modeling, Technical University of Denmark, March 2007.
- UCTE Operating handbook – Policies P1: load-frequency control and performance, July 2004.
- Wu F, Hou Y, Zhou H, Measuring reliability (or risk) coherently, Proceedings of the 16th Conference in Power Systems Computation – PSCC 2008, Glasgow, Scotland, July 14-18, 2008.